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Radiative Generation of the LMA Solution from Small Solar Neutrino Mixing at the GUT Scale

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Abstract

We show that in see-saw models with small or even vanishing lepton mixing angle θ_{12}, maximal θ_{23}, zero θ_{13} and zero CP phases at the GUT scale, the currently favored LMA solution of the solar neutrino problem can be obtained in a natural way by Renormalization Group effects. We find that most of the running takes place in the energy ranges above and between the see-saw scales. The Renormalization Group evolution of the solar mixing angle θ_{12} is generically much larger than the evolution of θ_{13} and θ_{23}. We present numerical examples of the evolution of the lepton mixing angles in the Standard Model and its Minimal Supersymmetric extension, the MSSM, in which the current best-fit values of the LMA mixing angles are produced with vanishing solar mixing angle θ_{12} at the GUT scale.

Key words: Renormalization Group Equation, Neutrino Masses, LMA Solution
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1 Introduction

To compare experimental results with predictions from models defined at a high energy scale, it is essential to evolve the parameters of the models from high to low energies by the Renormalization Group Equations (RGE's). At present, in the lepton sector the LMA solution of the solar neutrino problem with a large but non-maximal value of the solar mixing angle θ_{12} is strongly favored by the experiments [1–4]. On the other hand, the mixing angles in the quark sector are found to be small. It is therefore interesting to investigate if this discrepancy can be explained by the RG evolution of the lepton mixing angles, i.e. by increasing them via RG running.

The possibility of increasing a small atmospheric mixing angle θ_{23} by RG effects has been considered in [5,6]. The running of the solar mixing angle θ_{12}, starting at the mass scale of the heavy sterile neutrinos, was investigated in [7,8]. Other authors who studied the 3 neutrino case focused on nearly degenerate neutrinos [9–14], on the existence of fixed points [15,16], or on the effect of Majorana phases on the RG evolution of mixing angles [17].

We consider the see-saw scenario [18,19], i.e. the Standard Model (SM) or the Minimal Supersymmetric extension of the SM (MSSM) extended by 3 heavy neutrinos that are singlets under the SM gauge groups and have large explicit (Majorana) masses with a non-degenerate spectrum. Due to this non-degeneracy, one has to use several effective theories with the singlets partly integrated out when studying the evolution of the effective mass matrix of the light neutrinos [20,21]. Below the lowest mass threshold the neutrino mass matrix is given by the effective dimension 5 neutrino mass operator in the SM or MSSM. The relevant RGE's were derived in [21–26].

In a recent study [27], we showed that starting with bimaximal mixing at the GUT scale, the large solar mixing of the LMA solution can be explained as an effect of the RG evolution of the mixing angles. A key observation was that with bimaximal mixing and vanishing CP phases, the solar mixing angle changes considerably, while the evolution of the other angles is comparatively small.

In this paper, we assume arbitrary solar mixing θ_{12}, maximal atmospheric mixing θ_{23} and vanishing θ_{13} at the GUT scale. Especially interesting is the configuration with zero θ_{12}, which we examine in detail. In this study, we restrict ourselves to the case of vanishing CP phases and positive eigenvalues of the effective neutrino mass matrix at the GUT scale. We calculate the RG running numerically in order to obtain the mixing angles at low energy and to compare them with the experimentally favored values. Extending the analysis of [27], we find that the evolution of the solar mixing angle is much larger than the evolution of the other angles. Again, most of the running of the mixing angles takes place between and above the see-saw scales, which shows the importance of carefully studying the RG behavior in these regions. We derive analytic formulae, which help to understand this effect. The LMA solution of the solar neutrino problem can thus be obtained in a natural way from small or even vanishing solar mixing at the GUT scale.

2 Solving the RGE's

To evolve the lepton mixing angles and neutrino masses from the GUT scale to the electroweak (EW) or Supersymmetry (SUSY)-breaking scale in a see-saw model, a series of effective theories has to be used. These are obtained by successively integrating out the heavy singlets at their mass thresholds, which are non-degenerate in general. The derivation of the RGE's and the method for dealing with these effective theories are given in [21]. Starting at the GUT scale, the strategy is to solve the systems of coupled differential equations of the form

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\overset{(n)}{X}_i = \overset{(n)}{\beta}_{X_i}\left(\left\{\overset{(n)}{X}_j\right\}\right) \tag{1}$$

for all the parameters $\overset{(n)}{X}_i \in \left\{\overset{(n)}{\kappa}, \overset{(n)}{Y}_\nu, \overset{(n)}{M}, \ldots\right\}$ of the theory in the energy ranges corresponding to the effective theories denoted by (n). At each see-saw scale, tree-level matching is performed. Due to the complicated structure of the set of differential equations, the exact solution can only be obtained numerically. However, to understand certain features of the RG evolution, an analytic approximation at the GUT scale will be derived in section 2.2.

2.1 Initial Conditions at the GUT Scale

At the GUT scale M_{GUT}, we assume maximal atmospheric mixing θ_{23} and vanishing θ_{13}. We restrict ourselves to the case of positive mass eigenvalues and real parameters, so that there is no CP violation. In the basis where the charged lepton Yukawa matrix is diagonal, up to phase conventions the effective Majorana mass matrix of the light neutrinos is given by

$$m_\nu|_{M_{\mathrm{GUT}}} = V(\theta_{12}, 0, \tfrac{\pi}{4}) \cdot m_{\mathrm{diag}}|_{M_{\mathrm{GUT}}} \cdot V^T(\theta_{12}, 0, \tfrac{\pi}{4}) \;, \tag{2}$$

with $m_{\mathrm{diag}}|_{M_{\mathrm{GUT}}} := \mathrm{diag}\,(m_1|_{M_{\mathrm{GUT}}}, m_2|_{M_{\mathrm{GUT}}}, m_3|_{M_{\mathrm{GUT}}})$ and where

$$V(\theta_{12},\theta_{13},\theta_{23}) = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix} \tag{3}$$

with $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ is the (orthogonal) MNS matrix [28] in standard parametrization. In our see-saw scenario, the effective mass matrix of the light neutrinos is

$$m_\nu = \frac{v_{\mathrm{EW}}^2}{2}\, Y_\nu^T M^{-1} Y_\nu \tag{4}$$

at the high-energy scale, with $\langle\phi\rangle = \frac{v_{\mathrm{EW}}}{\sqrt{2}} \approx 174$ GeV. Obviously, the neutrino Yukawa matrix Y_ν and the singlet mass matrix M cannot be determined uniquely from this relation, i.e. there is a set of $\{Y_\nu, M\}$ configurations that yield single maximal mixing. After choosing an initial condition for Y_ν, M is fixed by the see-saw formula (4) if Y_ν is invertible. This determines the see-saw scales and thus the ranges of the various effective theories.

2.2 Analytic Calculations

The RGE for m_ν above the largest see-saw scale is given by

$$\begin{aligned}
16\pi^2\,\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_\nu &= C_e\left[Y_e^\dagger Y_e\right]^T m_\nu + C_e\, m_\nu\left[Y_e^\dagger Y_e\right] \\
&\quad + C_\nu\left[Y_\nu^\dagger Y_\nu\right]^T m_\nu + C_\nu\, m_\nu\left[Y_\nu^\dagger Y_\nu\right] \\
&\quad + \text{ terms with trivial flavour structure} \qquad (5)
\end{aligned}$$

with $C_e = -\frac{3}{2}$, $C_\nu = \frac{1}{2}$ in the SM and $C_e = C_\nu = 1$ in the MSSM. The terms with trivial flavor-structure do not influence the evolution of the mixing angles. To calculate the ratios of the RG evolution of the mixing angles at the GUT scale, we further use the parametrization

$$m_\nu(t) = V\big(\theta_{12}(t),\theta_{13}(t),\theta_{23}(t)\big)\cdot m_{\mathrm{diag}}(t)\cdot V^T\big(\theta_{12}(t),\theta_{13}(t),\theta_{23}(t)\big)\;, \qquad (6)$$

where μ is the renormalization scale, $t := \ln\frac{\mu}{\mu_0}$, and $m_{\mathrm{diag}} := \mathrm{diag}(m_1,m_2,m_3)$. As in [27], we can parametrize the neutrino Yukawa coupling Y_ν by

$$Y_\nu(y_1,y_2,y_3,\phi_{12},\phi_{13},\phi_{32}) = \mathrm{diag}(y_1,y_2,y_3)\cdot V^T(\phi_{12},\phi_{13},\phi_{32})\;. \qquad (7)$$

By differentiating equation (6) w.r.t. t and inserting the RGE (5), we obtain analytic expressions for $\dot\theta_{ij}$ and $\dot m_i$ at M_{GUT}, from which we can compute the rather lengthy expression for $\dot\theta_{12}/\dot\theta_{13}$ and $\dot\theta_{12}/\dot\theta_{23}$ at M_{GUT} as functions of the initial conditions. Inserting the initial condition $\theta_{13} = 0$ and $\theta_{23} = \frac{\pi}{4}$, we find that generically the RG evolution of θ_{12} is much larger than the change of the other angles, unless m_1 is very small.

For the special case $\theta_{12}|_{M_{\mathrm{GUT}}} = 0$, from these general formulae we obtain

$$\begin{aligned}
\left.\frac{\dot\theta_{12}}{\dot\theta_{13}}\right|_{M_{\mathrm{GUT}}} &= \frac{(m_2+m_1)\;(m_3-m_1)\;G_1}{(m_2-m_1)\;(m_3+m_1)\;G_2} \\
&\approx \begin{cases} \pm\dfrac{m_2+m_1}{m_2-m_1}\dfrac{G_1}{G_2} & \text{for hierarchical neutrino masses[1]} \\ \dfrac{\Delta m^2_{\mathrm{atm}}}{\Delta m^2_{\mathrm{sol}}}\dfrac{G_1}{G_2} & \text{for degenerate neutrino masses} \end{cases} \qquad (8a)
\end{aligned}$$

$$\begin{aligned}
\left.\frac{\dot\theta_{12}}{\dot\theta_{23}}\right|_{M_{\mathrm{GUT}}} &= \frac{(m_2+m_1)\;(m_3-m_2)\,G_1}{(m_2-m_1)\;(m_3+m_2)\;G_3} \\
&\approx \begin{cases} \pm\dfrac{m_2+m_1}{m_2-m_1}\dfrac{G_1}{G_3} & \text{for hierarchical neutrino masses[1]} \\ \dfrac{\Delta m^2_{\mathrm{atm}}}{\Delta m^2_{\mathrm{sol}}}\dfrac{G_1}{G_3} & \text{for degenerate neutrino masses} \end{cases} \qquad (8b)
\end{aligned}$$

where we neglected the contributions of Y_e and

$$\begin{aligned}
G_1 &= 8\,\cos(\phi_{12})\,\Big\{\big(2\,y_1^2-y_2^2-y_3^2\big)\,\cos(\phi_{13})\,[\sin(\phi_{13})-\cos(\phi_{13})\,\sin(\phi_{12})]\Big\} \\
&\quad + 4\,\cos(\phi_{12})\,\Big\{\big(y_2^2-y_3^2\big)\,\cos(2\,\phi_{23})\,[(3-\cos(2\,\phi_{13}))\,\sin(\phi_{12})+\sin(2\,\phi_{13})]\Big\} \\
&\quad + 8\,\big(y_2^2-y_3^2\big)\,(\cos(2\,\phi_{12})\,\sin(\phi_{13})-\cos(\phi_{13})\,\sin(\phi_{12}))\,\sin(2\,\phi_{23})\;, \qquad (9a)
\end{aligned}$$

[1] Note that this approximation is also valid for a relatively weak hierarchy, where m_3 is a few times larger or smaller than m_1, m_2.

$$
\begin{aligned}
G_2 \; = \; & -8\,\left(2\,y_1^2 - y_2^2 - y_3^2\right)\,\cos(\phi_{12})\,\cos(\phi_{13})\,\left(\cos(\phi_{13})\,\sin(\phi_{12}) + \sin(\phi_{13})\right) \\
& -4\,\left(y_2^2 - y_3^2\right)\,\cos(\phi_{12})\,\cos(2\,\phi_{23})\,\left[(\cos(2\,\phi_{13}) - 3)\,\sin(\phi_{12}) + \sin(2\,\phi_{13})\right] \\
& +8\,\left(y_2^2 - y_3^2\right)\,\left(\cos(\phi_{13})\,\sin(\phi_{12}) + \cos(2\,\phi_{12})\,\sin(\phi_{13})\right)\,\sin(2\,\phi_{23})\;, \qquad \text{(9b)} \\
G_3 \; = \; & \sqrt{2}\,\left(2\,y_1^2 - y_2^2 - y_3^2\right)\,\left[\cos^2(\phi_{12}) + (\cos(2\,\phi_{12}) - 3)\,\cos(2\,\phi_{13})\right] \\
& +\sqrt{2}\,\left(y_2^2 - y_3^2\right)\,\left[(\cos(2\,\phi_{12}) - 3)\,\cos(2\,\phi_{13}) - 6\,\cos^2(\phi_{12})\right]\,\cos(2\,\phi_{23}) \\
& +4\sqrt{2}\,\left(y_2^2 - y_3^2\right)\,\sin(2\,\phi_{12})\,\sin(\phi_{13})\,\sin(2\,\phi_{23})\;. \qquad \text{(9c)}
\end{aligned}
$$

Note that the relation $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}$ also holds at the GUT scale. Equations (8) and (9) can also be obtained from the formulae derived in [31]. The constants G_1, G_2 and G_3 clearly depend on the choice of $Y_\nu|_{M_{\text{GUT}}}$. However, unless the parameters $\{y_1, y_2, y_3, \phi_{12}, \phi_{13}, \phi_{32}\}$ are fine-tuned, we expect the ratios G_1/G_2 and G_1/G_3 to be of the order one. Thus, in the considered case of zero CP phases, the RG change of θ_{12} at the GUT scale is generically much larger than that of the other angles, unless m_1 is very small, in which case the ratio approaches 1.

3 Numerical Examples with Vanishing θ_{12} at the GUT Scale

In the previous section it was argued that the RG evolution of the solar mixing angle is generically larger than the evolution of the other mixing angles. This raises the question whether the LMA solution might be reached by RG evolution if one starts with vanishing θ_{12} and θ_{13} and maximal θ_{23} at M_{GUT}. An overview of the current allowed regions for the mixing angles and the mass squared differences is given in table 1.

	Best-fit value	Range (for $\theta_{ij} \in [0°, 45°]$)	C.L.
θ_{12} [°]	32.9	26.1 – 43.3	99% (3σ)
θ_{23} [°]	45.0	33.2 – 45.0	99% (3σ)
θ_{13} [°]	–	0.0 – 9.2	90% (2σ)
Δm^2_{sol} [eV^2]	$5 \cdot 10^{-5}$	$2.3 \cdot 10^{-5} - 3.7 \cdot 10^{-4}$	99% (3σ)
$\lvert\Delta m^2_{\text{atm}}\rvert$ [eV^2]	$2.5 \cdot 10^{-3}$	$1.2 \cdot 10^{-3} - 5 \cdot 10^{-3}$	99% (3σ)

Table 1: Experimental data for the neutrino mixing angles and mass squared differences. For the solar angle θ_{12} and the solar mass squared difference, the LMA solution has been assumed. The results stem from the analysis of the recent SNO data [3], the Super-Kamiokande atmospheric data [29] and the CHOOZ experiment [30].

We will investigate this possibility by numerical calculations in the following. We choose the specific form

$$
Y_\nu = X \cdot \begin{pmatrix} \varepsilon^2 & \varepsilon^3 & 0 \\ \varepsilon^3 & \varepsilon & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (10)
$$

for the neutrino Yukawa coupling at the GUT scale as an example. With a given Y_ν, M can be calculated from $m_\nu|_{M_{\text{GUT}}}$ by the see-saw formula (4).

3.1 Examples for the Running of the Lepton Mixing Angles

Figures 1 and 2 show numerical examples for the running of the lepton mixing angles from the GUT scale to the EW or SUSY-breaking scale, which produce the experimentally favored mixing angles of the LMA solution. The kinks in the plots correspond to the mass thresholds at the see-saw scales. The grey-shaded regions mark the various effective theories.

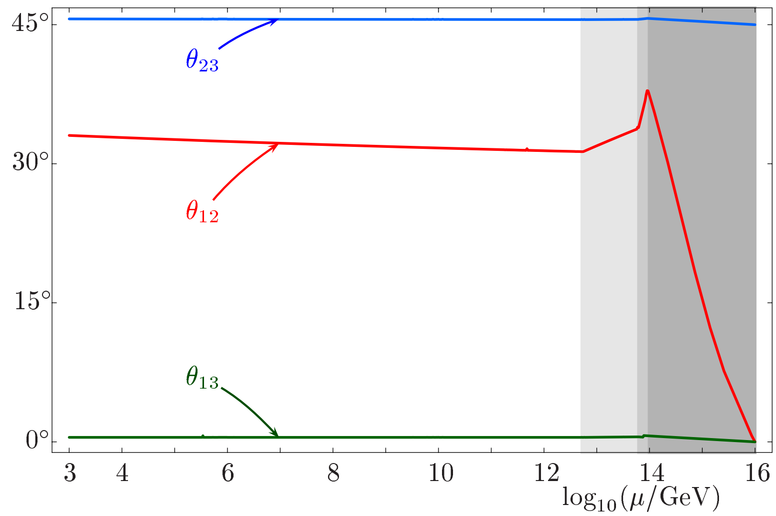


Figure 1: RG evolution of the mixing angles from the GUT scale to the SUSY-breaking scale (taken to be ≈ 1 TeV) in the MSSM extended by heavy singlets for a normal mass hierarchy with $\tan\beta = 5$, $\varepsilon = 0.65$, $m_1|_{M_{\mathrm{GUT}}} = 0.076\,\mathrm{eV}$ and $X = 0.5$. In this example, the lightest neutrino has a mass of $0.05\,\mathrm{eV}$.

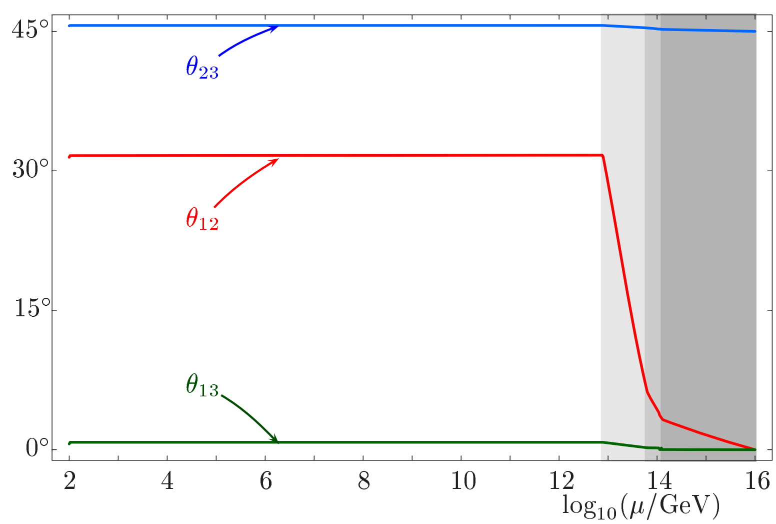
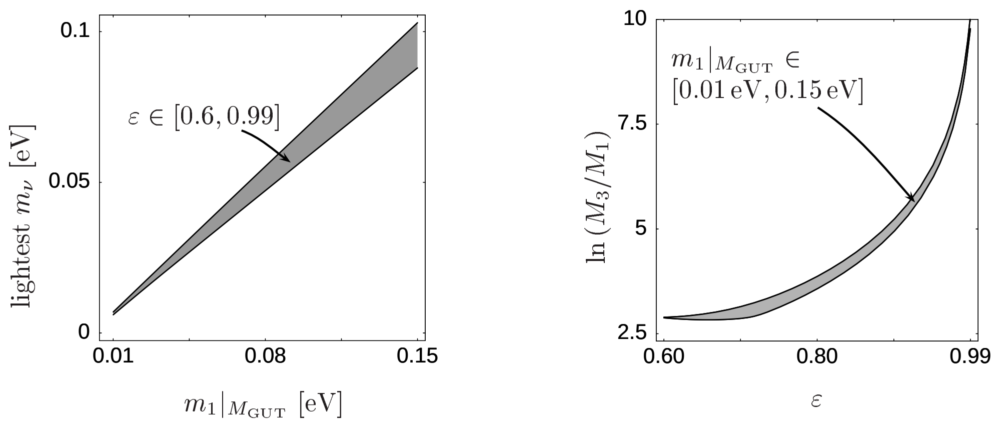


Figure 2: Example for the RG evolution of the mixing angles in the SM extended by heavy singlets from the GUT scale to the EW scale for a normal mass hierarchy with $\varepsilon = 0.6$, $m_1|_{M_{\mathrm{GUT}}} = 0.049\,\mathrm{eV}$ and $X = 0.5$. In this example, the lightest neutrino has a mass of $0.03\,\mathrm{eV}$. Most of the running takes place between the see-saw scales.

3.2 Parameter Space Regions Compatible with the LMA Solution

3.2.1 Interpretation of the Parameters at the GUT Scale

The parameter ε introduced in equation (10) controls the hierarchy of the entries in Y_ν and thus the degeneracy of the see-saw scales. Moreover, we choose the lightest neutrino mass at the GUT scale, $m_1|_{M_{\rm GUT}}$ for a regular and $m_3|_{M_{\rm GUT}}$ for an inverted spectrum as a further initial condition. We fix the GUT scale values of the two remaining masses by the requirement that the solar and atmospheric mass squared differences obtained at the EW scale after the RG evolution be compatible with the allowed experimental regions. Thus, we are left with the free parameters X, ε and $m_1|_{M_{\rm GUT}}$ or $m_3|_{M_{\rm GUT}}$. The dependence of the degeneracy of the see-saw scales on ε and the mass of the lightest neutrino at the low scale from its initial value at the GUT scale is shown in figure 3. As mentioned above, we work in the basis where the Yukawa matrix of the charged leptons is diagonal.



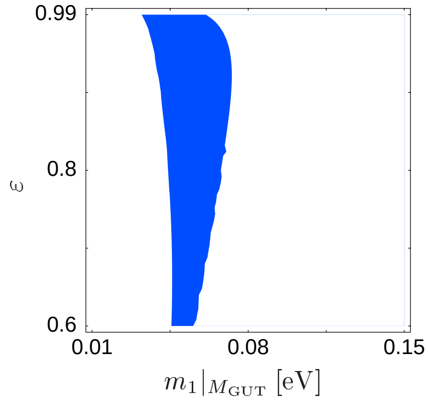
(a) Parameter $m_1|_{M_{\rm GUT}}$ (b) Parameter ε

Figure 3: Plot 3(a) shows the mass of the lightest neutrino (at low energy) as a function of $m_1|_{M_{\rm GUT}}$ for the SM and the MSSM with normal mass hierarchy, $X = 0.5$ and $\varepsilon \in [0.6, 0.99]$ (grey region). Plot 3(b) shows the degeneracy of the see-saw scales, parametrized by $\ln(M_3/M_1)$ (at the GUT scale), as a function of ε for the same cases with $m_1|_{M_{\rm GUT}} \in [0.01\ \text{eV}, 0.15\ \text{eV}]$ (grey region).

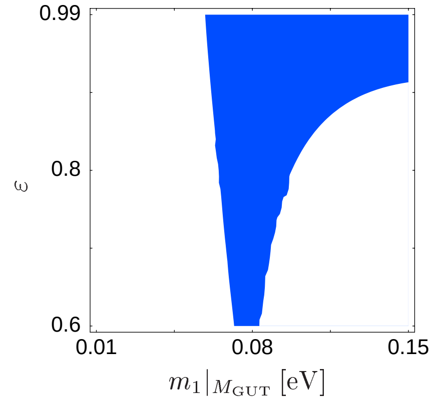
3.2.2 Allowed Parameter Space Regions

The parameter space regions in which the RG evolution produces low-energy values compatible with the LMA solution are shown in figure 4 for the SM and the MSSM ($\tan\beta = 5$) with a normal mass hierarchy and Y_ν given in equation (10).

We would like to stress that the shape of the allowed parameter space regions strongly depends on the choice of the initial value of Y_ν at the GUT scale. For inverted neutrino mass spectra in the MSSM, allowed parameter space regions exist as well. For the chosen range of initial conditions at the GUT scale as in figure 4, allowed regions with an inverted neutrino mass spectrum do not exist in the SM. One also has to ensure that the sign of $\Delta m^2_{\rm sol}$ is positive, as the LMA solution requires this if the convention is used



(a) SM



(b) MSSM

Figure 4: Parameter space regions compatible with the LMA solution of the solar neutrino problem for the example with Y_ν given by equation (10). The initial condition at the GUT scale $M_{\rm GUT} = 10^{16}$ GeV is vanishing mixing for θ_{12} and θ_{13} and maximal mixing for θ_{23}. The comparison with the experimental data is performed at the EW scale or at 1 TeV for the SM and the MSSM, respectively. The white regions of the plots are excluded by the data (LMA) at 3σ. For this example, we consider the case of a normal neutrino mass hierarchy and $X = 0.5$ for the scale factor of the neutrino Yukawa couplings.

that the solar mixing angle is smaller than 45°. For the chosen range of initial conditions there also exist regions, where the evolution of the solar mixing angle is too large, i.e. the mixing angle would run above 45°. This would correspond to a negative $\Delta m^2_{\rm sol}$ and thus these regions have to be excluded.

4 Summary and Conclusions

We have studied the RG evolution of the lepton mixing angles in see-saw scenarios in the SM and in the MSSM. We have shown that the experimentally favored neutrino mass parameters of the LMA solution can be obtained in a rather generic way from initial conditions with small or even zero solar mixing θ_{12}, maximal θ_{23} and zero θ_{13} at the GUT scale. We have concentrated on the case of vanishing CP phases, which implies positive mass eigenvalues[3]. We found that generically the RG evolution of the solar mixing angle is enhanced compared to the RG change of the two other mixings if one starts with maximal θ_{23}, zero θ_{13} and arbitrary solar mixing θ_{12} at the GUT scale. For the special case of bimaximal GUT scale mixing, this has recently been observed in [27]. In the SM and MSSM with $\theta_{12} = \theta_{13} = 0°$ and $\theta_{23} = 45°$ as initial conditions at $M_{\rm GUT}$, we have found regions in parameter space in which mixing angles compatible with the experimentally favored values of the LMA solution are produced. The effect does not require fine-tuning, degenerate neutrinos or a large value of $\tan\beta$ (in the case of the MSSM). This opens up new possibilities for building models of neutrino masses.

[3]The general case of arbitrary CP phases is beyond the scope of this study and will be investigated in a forthcoming paper [32].

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References

[1] V. Barger, D. Marfatia, K. Whisnant, and B. P. Wood, *Imprint of SNO neutral current data on the solar neutrino problem*, Phys. Lett. **B537** (2002), 179–186 (`hep-ph/0204253`).

[2] A. Bandyopadhyay, S. Choubey, S. Goswami and D. P. Roy, *Implications of the first neutral current data from SNO for solar neutrino oscillation*, Phys. Lett. **B540** (2002), 14 (`hep-ph/0204286`).

[3] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Peña-Garay, *Before and After: How has the SNO NC measurement changed things?*, JHEP **0207** (2002), 054 (`hep-ph/0204314`).

[4] P. C. de Holanda and A. Yu. Smirnov, *Solar neutrinos: Global analysis with day and night spectra from SNO*, `hep-ph/0205241`.

[5] M. Tanimoto, *Renormalization effect on large neutrino flavor mixing in the minimal supersymmetric standard model*, Phys. Lett. **B360** (1995), 41–46 (`hep-ph/9508247`).

[6] K. R. S. Balaji, A. S. Dighe, R. N. Mohapatra and M. K. Parida, *Radiative magnification of neutrino mixings and a natural explanation of the neutrino anomalies*, Phys. Lett. **B481** (2000), 33–38 (`hep-ph/0002177`).

[7] T. Miura, E. Takasugi and M. Yoshimura, *Quantum effects for the neutrino mixing matrix in the democratic-type model*, Prog. Theor. Phys. **104** (2000), 1173–1187 (`hep-ph/0007066`).

[8] T. Miura, T. Shindou and E. Takasugi, *Exploring the neutrino mass matrix at M(R) scale*, hep-ph/0206207.

[9] J. R. Ellis and S. Lola, *Can neutrinos be degenerate in mass?*, Phys. Lett. **B458** (1999), 310–321 (`hep-ph/9904279`).

[10] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, *Naturalness of nearly degenerate neutrinos*, Nucl. Phys. **B556** (1999), 3–22 (`hep-ph/9904395`).

[11] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, *Nearly degenerate neutrinos, supersymmetry and radiative corrections*, Nucl. Phys. **B569** (2000), 82–106 (`hep-ph/9905381`).

[12] P. H. Chankowski, A. Ioannisian, S. Pokorski and J. W. F. Valle, *Neutrino unification*, Phys. Rev. Lett. **86** (2001), 3488–3491 (hep-ph/0011150).

[13] M.-C. Chen and K. T. Mahanthappa, *Implications of the renormalization group equations in three neutrino models with two-fold degeneracy*, Int. J. Mod. Phys. **A16** (2001), 3923–3930 (hep-ph/0102215).

[14] M. K. Parida, C. R. Das and G. Rajasekaran, *Radiative stability of neutrino-mass textures*, hep-ph/0203097.

[15] P. H. Chankowski, W. Krolikowski and S. Pokorski, *Fixed points in the evolution of neutrino mixings*, Phys. Lett. **B473** (2000), 109 (hep-ph/9910231).

[16] G. Dutta, *Stable bimaximal neutrino mixing pattern*, hep-ph/0203222.

[17] N. Haba, Y. Matsui and N. Okamura, *The effects of Majorana phases in three-generation neutrinos*, Eur. Phys. J. **C17** (2000), 513–520 (hep-ph/0005075).

[18] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity* (P. van Nieuwenhuizen and D. Z. Freedman, eds.), North Holland, Amsterdam, 1979, p. 315.

[19] T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe* (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95.

[20] S. F. King and N. N. Singh, *Renormalisation group analysis of single right-handed neutrino dominance*, Nucl. Phys. **B591** (2000), 3–25 (hep-ph/0006229).

[21] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass matrix running for non-degenerate see-saw scales*, Phys. Lett. **B538** (2002), 87–95 (hep-ph/0203233).

[22] P. H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw neutrino masses*, Phys. Lett. **B316** (1993), 312 (hep-ph/9306333).

[23] K. S. Babu, C. N. Leung and J. Pantaleone, *Renormalization of the neutrino mass operator*, Phys. Lett. **B319** (1993), 191 (hep-ph/9309223).

[24] S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass operator renormalization revisited*, Phys. Lett. **B519** (2001), 238–242 (hep-ph/0108005).

[25] S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass operator renormalization in two Higgs doublet models and the MSSM*, Phys. Lett. **B525** (2002), 130 (hep-ph/0110366).

[26] S. Antusch and M. Ratz, *Supergraph techniques and two-loop beta-functions for renormalizable and non-renormalizable operators*, JHEP **0207** (2002), 059 (hep-ph/0203027).

[27] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *The LMA solution from bimaximal lepton mixing at the GUT scale by renormalization group running*, hep-ph/0206078.

[28] Z. Maki, M. Nakagawa and S. Sakata, *Remarks On The Unified Model Of Elementary Particles*, Prog. Theor. Phys. **28** (1962), 870.

[29] T. Toshito *et al.* [Super-Kamiokande Collaboration], *Super-Kamiokande atmospheric neutrino results*, `hep-ex/0105023`.

[30] M. Apollonio *et al.* [CHOOZ Collaboration], *Limits on neutrino oscillations from the CHOOZ experiment*, Phys. Lett. **B466** (1999), 415 (`hep-ex/9907037`).

[31] J. A. Casas, J. R. Espinosa, A. Ibarra and I. Navarro, *General RG equations for physical neutrino parameters and their phenomenological implications*, Nucl. Phys. **B573** (2000), 652 (`hep-ph/9910420`).

[32] S. Antusch, J. Kersten, M. Lindner and M. Ratz, *In preparation.*